# ATP Binding Enables Broad Antibiotic Selectivity of Aminoglycoside Phosphotransferase ( $\mathbf{3}^{\prime}$ )-IIIa - An Elastic Network Analysis 

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# Supplemental Data 

## Derivation of eq 10

For a diagonalisable $n \times n$ square matrix $A$, denote the pseudo-inverse by $\tilde{A}^{-1}$. If $A$ is invertible, then of course $\tilde{A}^{-1}=A^{-1}$. Denote the unit vectors by $e_{i}$ and the vector with all entries 1 by $e$.

Let $\lambda_{1} \leq \ldots \leq \lambda_{n}$ be the eigenvalues of $A, \Lambda=\operatorname{diag}\left(\lambda_{1}, \ldots, \lambda_{n}\right)$, and $U$ the orthogonal matrix with rows given by the normalized eigenvectors of $A$. Then

$$
\begin{equation*}
A=U^{\mathrm{T}} \Lambda U \quad \text { and } \quad \tilde{A}^{-1}=U^{\mathrm{T}} \tilde{\Lambda}^{-1} U \tag{1}
\end{equation*}
$$

Further let $v=\left(v_{1}, \ldots, v_{n}\right)^{\mathrm{T}}=U e_{i}$, and note that $v$ is a unit vector, $\sum_{k} v_{k}^{2}=1$. Then

$$
\begin{equation*}
A_{i i}=e_{i} \cdot A e_{i}=U e_{i} \cdot \Lambda U e_{i}=v \cdot \Lambda v=\sum_{k=1}^{n} \lambda_{k} v_{k}^{2} \tag{2}
\end{equation*}
$$

If $A$ is invertible,

$$
\begin{equation*}
A_{i i}^{-1}=U e_{i} \cdot \Lambda^{-1} U e_{i}=\sum_{k=1}^{n} v_{k}^{2} \frac{1}{\lambda_{k}}=\frac{1}{A_{i i}} \sum_{k, l=1}^{n} v_{k}^{2} v_{l}^{2} \frac{\lambda_{l}}{\lambda_{k}} \tag{3}
\end{equation*}
$$

Now we use the fact that $\frac{a}{b}+\frac{b}{a} \geq 2$ for any strictly positive reals $a, b$, and obtain

$$
\begin{align*}
\sum_{k, l=1}^{n} v_{k}^{2} v_{l}^{2} \frac{\lambda_{l}}{\lambda_{k}} & =\sum_{k<l} v_{k}^{2} v_{l}^{2}\left(\frac{\lambda_{l}}{\lambda_{k}}+\frac{\lambda_{k}}{\lambda_{l}}\right)+\sum_{k} v_{k}^{4} \geq 2 \sum_{k<l} v_{k}^{2} v_{l}^{2}+\sum_{k} v_{k}^{4} \\
& =\sum_{k, l} v_{k}^{2} v_{l}^{2}=\sum_{k} v_{k}^{2} \sum_{l} v_{l}^{2}=1 \tag{4}
\end{align*}
$$

Therefore, if $A$ is invertible, it follows that

$$
\begin{equation*}
\tilde{A}_{i i}^{-1} \geq \frac{1}{A_{i i}} \tag{5}
\end{equation*}
$$

The Kirchhoff matrix is a symmetric, positive semi-definite matrix. The eigenvalue 0 has multiplicity 1 and corresponding eigenvector $e$. We obtain in the same way as in Equations 3 and 4

$$
\begin{equation*}
\tilde{A}_{i i}^{-1}=\frac{1}{A_{i i}} \sum_{k, l=2}^{n} v_{k}^{2} v_{l}^{2} \frac{\lambda_{l}}{\lambda_{k}} \geq \frac{1}{A_{i i}} \sum_{k=2}^{n} v_{k}^{2} \sum_{l=2}^{n} v_{l}^{2}=\frac{1}{A_{i i}}\left(1-v_{1}^{2}\right)^{2} . \tag{6}
\end{equation*}
$$

As $e$ is an eigenvector for $\lambda_{1}$,

$$
\begin{equation*}
v_{1}^{2}=\frac{\left(e_{i} \cdot e\right)^{2}}{|e|^{2}}=\frac{1}{n}, \tag{7}
\end{equation*}
$$

and it follows

$$
\begin{equation*}
\tilde{A}_{i i}^{-1} \geq \frac{1}{A_{i i}}\left(\frac{n-1}{n}\right)^{2} \tag{8}
\end{equation*}
$$

(a)

(b)


Figure S 1: Difference correlation plot showing the deviation from additivity of the effects of substrate binding. The correlation differences of $X$ [APH-Nuc-Kana] minus $M_{\mathrm{kan}}[\mathrm{APH}]$ are subtracted from the sum of the single effects, i.e. the difference correlations of $X[\mathrm{APH}-\mathrm{Nuc}-$ Kana] minus $M_{\text {kan }}$ [APH-Kana] added to the difference correlations of $X$ [APH-Nuc-Kana] minus $M_{\text {kan }}[$ APH-Nuc]. The highest absolute value of deviation is $<0.05$. (a) Color scale corresponds to color scale of Figure 2b,c,d. (b) Using a color scale ranging from -0.05 to +0.05 shows that small deviations from additivity occur in the regions affected by substrate, especially nucleotidebinding.


Figure S 2: Correlated motions of APH calculated with the ANM for X[APH-Nuc-Kana], using a cutoff radius of $10 \AA$. While the classification into three protein domains is less obvious than from GNM calculations, the substrate-dependent correlation differences are very similar to the differences obtained with the GNM. (a) Correlation plot of $X$ [APH-Nuc-Kana]. (b) The difference correlation plot of original $X$ [APH-Nuc-Kana] complex minus correlations of model complex $M_{\text {kan }}$ [APH-Kana] shows the effect of nucleotide binding to the binary APH-kanamycin complex. (c) The difference correlation plot of original $X$ [APH-Nuc-Kana] complex minus correlations of model complex $M_{\text {kan }}[\mathrm{APH}-\mathrm{Nuc}]$ shows the effect of kanamycin binding to the binary APH-nucleotide complex. (d) The difference correlation plot of original X[APH-Nuc-Kana] complex minus correlations of model complex $M_{\text {kan }}[\mathrm{APH}]$ shows that the effects of binding of both substrates are nearly additive.

Table S 1: Classification of nodes in dynamic domains calculated for the different APH structures.

|  | Domain I | Domain II | Domain III |
| :--- | :---: | :---: | :---: |
| $X[\mathrm{APH}]$ | $5-91$ | $92-129,180-249$ | $130-179,250-264$ |
| $M_{\text {nuc }}[\mathrm{APH}]$ | $2-95,197-202$ | $96-136,180-249$ | $137-179,250-264$ |
| $M_{\mathrm{kan}}[\mathrm{APH}]$ | $2-95$ | $96-136,180-255$ | $137-179,256-264$ |
| $M_{\text {neo }}[\mathrm{APH}]$ | $2-95$ | $96-136,180-255$ | $137-179,253-264$ |

Table S 2: Comparison between experimental and theoretical B-factors for different structures and cutoff distances $d_{\text {cut }}$. The linear correlation coefficient $\rho$ between experimental B-factors $\left(x_{i}\right)$ and calculated B-factors $\left(y_{i}\right)$ is given by $\rho=\frac{\sum\left(x_{i}-x\right)\left(y_{i}-y\right)}{\sqrt{\sum\left(x_{i}-x\right)^{2} \sum\left(y_{i}-y\right)^{2}}}$. $x$ and $y$ are the mean values of the corresponding B-factors.

|  | 1J7I | 1J7U | 1L8T | 2B0Q |
| :--- | :---: | :---: | :---: | :---: |
| Substrate(s) | - | MgAMPPNP | MgADP, kanamycin A | MgADP, neomycin B |
| $\rho\left(d_{\text {cut }}=6 \AA\right)$ | 0.14 | 0.32 | 0.15 | 0.38 |
| $\rho\left(d_{\text {cut }}=7 \AA\right)$ | 0.26 | 0.54 | 0.48 | 0.47 |
| $\rho\left(d_{\text {cut }}=8 \AA\right)$ | 0.25 | 0.55 | 0.55 | 0.50 |
| $\rho\left(d_{\text {cut }}=9 \AA\right)$ | 0.29 | 0.51 | 0.57 | 0.49 |



Figure S 3: Correlation change upon binding of a pseudo-substrate to $X[\mathrm{APH}]$. The correlations of the structure with pseudo-substrate are subtracted from the correlations of $X[\mathrm{APH}]$. The pseudo-substrates are either bound on the surface of one domain (first row), or at the interface between two domains (second and third row). Most pseudo-substrates have only minor and very localized effects on the correlations. Only the two pseudo-substrates lying between domain I and III have a large effect on the correlations, which is comparable to the effect of real ligand binding.


Figure S 4: NMR H/D exchange times (blue curve) and theoretical B-factors (red curve) for the antibiotic complexes of APH. (a) $M_{\text {kan }}[\mathrm{APH}-\mathrm{Kana}]$ and (b) $M_{\text {neo }}[\mathrm{APH}-\mathrm{Neo}]$ are used for the calculations. The theoretical B-factors are shifted such that the lowest B-factor of each structure is zero. Peaks never exchanged in 96 hours are cut off at 54.0 hr line and 51.5 hr line. A time value of zero means that the hydrogen exchange occurred faster than the start of the acquisition of the first spectrum (3-4 min of delay to start data acquisition). Exchange times are only measured for about half of the residues. Generally, long H/D exchange times correspond to low B-factors, and vice versa.



Figure S 5: Correlated motion of APH in binary complexes with kanamcyin and neomycin. (a) Correlation plot of $M_{\text {kan }}[\mathrm{APH}-\mathrm{Kana}]$. (b) Correlation plot of $M_{\text {neo }}$ [APH-Neo]. The positive correlations of residues 157 to 162 of domain III to each other and to the C-terminal residues is higher in the neomycin-bound form. The correlations between residues 157 to 162 of domain III and residues 226 to 230 of domain II are approx. zero in $M_{\text {kan }}[A P H-K a n a]$, because they are strongly connected over kanamycin. With neomycin, the two stretches are anticorrelated. (c) Correlation differences between $M_{\text {kan }}[\mathrm{APH}]$ and $M_{\text {neo }}[\mathrm{APH}]$ arising from structural differences between $X$ [APH-Nuc-Kana] and $X[$ APH-Nuc-Neo]. The correlation differences of residues 157 to 162 in plots a and b do not arise from structural differences, but are due to binding of the antibiotic.

